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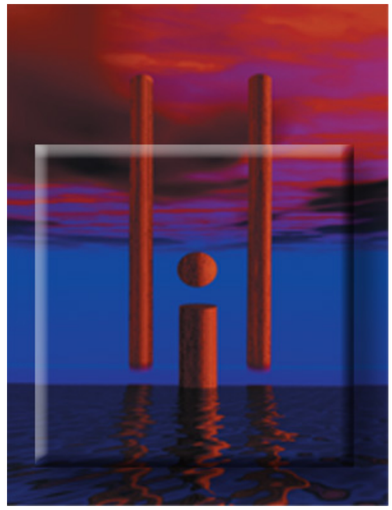
Productivity Growth in Canada



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Appendix 1 – The Statistics Canada Productivity Program: Concepts and Methods

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A.1 Introduction

This appendix describes the concepts and methods underlying Statistics Canada's indices of productivity growth. Its primary objective is to provide an accessible guide to the various productivity measures produced by Statistics Canada within a coherent framework that strikes a balance between theoretically desirable characteristics of productivity measures and the reality of data availability. A second objective is to indicate how Statistics Canada's productivity measures compare with those produced by the U.S. Bureau of Labor Statistics and the Organisation for Economic Co-operation and Development (OECD) for cross-country comparison purposes. Finally, the appendix provides comments on some of the conceptual and empirical obstacles to further improvements in the measure.

The publication of productivity measures has long been an important activity of Statistics Canada. This measurement program has evolved over the years, stimulated by changes in data availability, by new developments in the economics literature, and also by the needs of data users. Following the development of the Canadian System of National Accounts (CSNA) after the Second World War, Statistics Canada introduced labour productivity measures for the aggregate business sector and its major constituent subsectors.[1] More recently, the agency has developed measures of multifactor productivity. These measures, which consider the productivity of a bundle of inputs (labour, capital, and purchased goods and services[2]), are often used as 'red flags' to measure the extent to which economic performance differs across industries, across countries and over time.

Statistics Canada's productivity program has the following characteristics often shared by those of other statistical offices. First, it focuses exclusively on comparisons based on productivity growth measures as opposed to productivity levels. At present, rates of change are preferred because they avoid methodological and data problems associated with productivity level comparisons. Second, the program produces various kinds of productivity measures of the business sector and its major constituents (subsectors and industries).

[1] The definition of business sector used for productivity measures excludes all non-commercial activities as well as the rental value of owner-occupied dwellings. Corresponding exclusions are also made to the inputs. Business gross domestic product (GDP), as defined by the productivity program, represents 71% of the economy GDP in 1992. The business sector is split into the following major subsectors: goods-producing, services and manufacturing. The goods-producing subsector consists of agriculture, fishing, forestry, mining, manufacturing, construction and public utilities. Services comprise transportation and storage, communications, wholesale and retail trade, finance, insurance and real estate, and the group of community, business and personal services.

[2] Purchased goods and services are known as intermediate inputs in the CSNA.

A.2 Theory and concepts

A.2.1 Productivity measures

Productivity growth is commonly defined as the difference between the percentage change of a measure of output and the percentage change of a measure of inputs used. It is meant to capture the growth in productive efficiency arising from technical progress. Productivity growth is the growth of output not accounted for by the growth of an input or inputs.

There are various productivity growth measures. The choice between them depends on the purpose of productivity measurement and, in many instances, on data availability. In general, productivity measures can be grouped into two broad categories:

1. The first is single-factor productivity where growth in output is compared with growth of input. The most commonly used single-factor productivity measure is labour productivity (LP) growth , measured as:

$$\Delta\, LP = \Delta\, Q - \Delta\, L\,, \tag{1}$$

   where Δ refers to discrete changes in percentage with respect to time; Q and L represent, respectively, output and labour.

   Although labour productivity growth is an important measure, it is not the only way to measure gains in productive efficiency. Economic performance as measured by labour productivity must be interpreted carefully, since these estimates reflect changes in the other inputs (e.g., capital) in addition to growth in productive efficiency. The production of output requires the combination of all inputs in a technologically feasible manner. Hence, productivity is also measured in a way that compares output with the combined use of all resources, not just labour. For example, the construction of a complex plant with substantial expenditures on capital equipment but only minimal operating expenditures for labour may generate an apparently impressive labour productivity index, but the total amortized capital, plus labour cost may be much higher than those of a less complex but slightly more labour-intensive plant that would be more efficient while yielding a smaller labour productivity index. For these reasons, caution is in order in the interpretation of either rapid gains or 'disturbing slowdowns' in labour productivity growth. This sentiment is shared, incidentally, by both labour economists and productivity analysts (Griliches 1980; Rees 1980).

2. Users are therefore encouraged to consider a second way of measuring productivity growth, one that complements labour productivity growth. This second measure is known as multifactor productivity growth (MFP), the difference in the growth in output (Q) minus the growth in a bundle of inputs (I):

$$\Delta\, MFP = \Delta\, Q - \Delta\, I\,. \tag{2}$$

   Multifactor productivity growth is often characterized as arising from an outward shift in the production function resulting from technical progress. The concept of multifactor productivity, developed by Solow (1958), depends, for the sake of simplicity, upon the assumptions of constant returns to scale, perfect adjustments to the inputs and competitive markets. It measures technical progress as a residual; that is, the growth of the output is not due to the growth of the inputs. But Solow

Table A1.1 Most commonly used concepts of productivity

Concept of inputs	Concept of output	
	Gross output	Value added
Labour	–	Labour productivity
Capital	–	–
Combined capital and labour	–	Multifactor productivity
Combined capital, labour, energy, materials and services	Multifactor productivity	–

also acknowledged that multifactor productivity so measured reflects many other influences, because it is calculated as a residual.

Other research has made contributions facilitating the implementation of the multifactor productivity framework by statistical agencies. Domar (1961) demonstrated how a system of industry and aggregate production functions could be used to produce a set of industry productivity measures that are consistent with the aggregate measures for the economy as a whole. Jorgenson and Griliches (1967) showed how detailed data could be used to construct a capital aggregate without making strong assumptions about the relative marginal products of dissimilar assets. Also, it was recognized that fixed-based formulas could introduce bias into the aggregating process. Diewert (1976) showed how production functions could be used to provide a basis for determining which index number formulas were least restrictive. He developed a number of arguments detailing the attractive properties of superlative indices.

Measures of productivity differ partly because of the comprehensiveness of inputs covered. They also differ in terms of the measure of output used. There are two major distinctions—whether output is measured by value added or by gross final output. Table A1.1 lists a variety of single-factor and multifactor productivity concepts that are generally used for different analytical purposes. In the first case, the bundle of inputs consists of labour and capital. In the second case, it consists of labour, capital, energy, materials, and services.

A.2.2 Output and inputs

A.2.2.1 Output current prices

The information needed for the measurement of production activity is drawn from the income statement of individual businesses. In the income statement, revenues come mainly from sales; costs of goods and services sold include mainly purchased goods and services and labour compensation (wages and salaries and supplementary labour income).

Rearranged and modified, the income statement for the business unit provides the production account that constitutes the starting point for deriving the input-output accounts of an industry. The production account, derived from the income statement through some suitable modifications,[3] records the production attributable to the business unit in

[3] These modifications are necessary because sales (shown in the income statement) are not equal to the value of production. Sales are not equivalent to gross output because the business unit may either make sales from inventories of finished goods produced in previous periods or place current production in inventories. Thus, gross output is obtained as the sum of sales and the value of changes in inventories.

Table A1.2 Production account of producing units A1 and A2

Uses		Resources		
Producing unit A1				
Labour compensation	380	Gross output		+1,000
Surplus or compensation of capital	+120			
		Producing unit A1	120	
		Producing unit A2	+300	
		Industry B	+ 80	
		Purchased goods and services		-500
Charges against output	500	Value added		500
Producing unit A2				
Labour compensation	150	Gross output		300
Surplus or compensation of capital	+ 50			
		Producing unit A1	50	
		Producing unit A2	+ 0	
		Industry B	+50	
		Purchased goods and services		-100
Charges against output	200	Value added		200

Table A1.3 Production account of industry A (consolidation of producing units A1 and A2)

Uses		Resources		
Labour compensation	530	Gross output		1,300
Surplus or compensation of capital	+170	Intra-industry flows of goods and services	-470	
		Gross output net of intra-industry transactions		830
		Purchased goods and services (industry B)	-130	
Charges against output	700	Value added		700

terms of both goods and services produced and the income payments and other costs arising in production.

For the sake of an illustration, consider a business sector with two industries A and B, where A comprises two producing units A1 and A2. Table A1.2 displays the production accounts of these two units. For example, to produce $1,000 of output, the unit A1 consumes a portion of its own output ($120), a portion of the output produced by industry B ($80) and the whole output of the unit A2 ($300); it also hires employees who are paid $380. Once the employees and the purchased goods and services have been paid, the unit A1 is left with a residual of $120 to compensate the owners of capital.

The production account gives rise to two concepts of output. The first is value added, which is the sum of compensation of the primary inputs—labour and capital; this is also known as gross domestic product (GDP). The second is gross output, which is the sum of value added and the value of purchased goods and services. Value added constitutes an unduplicated measure of output. In addition, the sum of value added across all producing units is invariant to the degree of vertical integration between those units. In that sense, value added is perfectly additive. Table A1.3, which consolidates the information of the production units A1 and A2, shows that value added remains the same. By contrast, gross output suffers from double counting as the value of purchased goods and services by a unit has already been counted as output of another unit and the consolidation of producing units will change the measure of gross output.

Different measures of output are adopted by productivity practioners, depending on how they treat those transactions that occur within industry A (the consolidation of units A1 and A2), i.e., intra-industry deliveries of intermediate inputs. If the producing units A1 and A2 were integrated together into a single consolidated 'establishment' covering the whole industry A, then intra-industry purchases are netted out and gross output is then defined net of intra-industry transactions.[4] The production accounts of producing units A1 and A2 indicate that the inclusion of intra-industry flows of purchased goods and services adds identically to both the input and output side of industry A's production account, as the value of gross output and the value of purchased goods and services change with the exclusion of intra-industry transactions (Table A1.3).

The process of vertical integration may be pushed one step further to cover not only intra-industry sales but also inter-industry sales. The establishments of an industry may be integrated with their upstream suppliers, which may themselves be integrated upstream with their own suppliers. The associated concept of output in this case is called inter-industry output as it takes into account the inter-industry transactions (Rymes 1972; Wolfe 1991; Durand 1996). Under full integration, the output of industries becomes a function of the direct use of the industries' own primary inputs and the indirect use of the primary inputs of all upstream suppliers.

Constant prices

Productivity measures require estimates of real output produced and real inputs used in the production process. This is done by estimating the value of output and inputs in constant prices. The notion of constant prices is not one that can be defined in terms of physical units of output and inputs. There is no meaningful way to tally up, on a common physical unit of measurement, the diverse range of goods and services found in the economy. Rather, the aggregation is performed in monetary terms as the value, at fixed prices, of the goods and services included in the output and inputs.

The technique employed for deriving constant price series of value added is known as the 'double deflation' method. This involves deflating the gross output and the intermediate inputs separately and subtracting one from the other. This derivation of industry real output circumvents the problem of deflating the compensation of primary inputs, an alternative that could be used.

A.2.2.2 Inputs

Labour input

Over time the composition of the labour force has changed significantly in Canada, as in many other developed countries: more jobs are non-standard (part-time, temporary and self-employed); the distribution of hours worked has become more polarized (the number of persons working both short and long hours has steadily increased over the last two decades). If labour is measured in terms of number of employees, no consideration is given to the fact that some employees work a standard workweek and others do not. Measuring labour input as the number of hours worked deals with this aspect of heterogeneous labour input.

[4] This concept of output net of intra-industry transactions is also known as sector output (Gollop 1979).

Labour also varies considerably in terms of quality. For example, education has been increasing. Measuring labour input may be done either via simple aggregates or by aggregating different types of labour using different weights, based on their relative wage rate. The former ignores differences in quality. The latter adjusts for quality differentials by assuming that they are reflected in relative wage rates.

Capital input

Capital input shares some of the same characteristics as labour input. Capital goods purchased or rented by a firm also constitute repositories of capital services, much like employees hired for a certain period of time who can be seen as carriers of human capital and, therefore, as repositories of labour services. There is, however, an important difference between labour and capital: except for rented capital, no market transaction is actually recorded when capital provides services to its user. Therefore, unlike labour, no explicit price and quantity of the service rendered can be observed for capital. An implicit measure of the price of capital services, derived from the ratio of capital compensation to the stock of capital, captures the internal rate of return used in the cost of capital formula. This measure, which varies only across industries, is used to construct capital services at the level of the business sector or its subsectors (such as manufacturing and services).

As with labour, measures of capital growth can be made as simple aggregates across capital types (machinery versus buildings) or by weighting the different asset classes by weights that reflect differences in the capital services yielded by a dollar of assets in each category.

Intermediate inputs

Estimates of intermediate inputs such as energy, materials and services in current and constant prices are required for the construction of gross output, value added and, ultimately, multifactor productivity series. The weighted sum of the growth rates of intermediate inputs in constant prices enters into the calculation of a) value added in constant prices (double deflation technique) and b) multifactor productivity estimates based on gross output. The weights of intermediate inputs are defined as the ratio of the value of each intermediate input to gross output in current prices.

A.3 Measurement framework

A.3.1 Productivity measures at Statistics Canada

Statistics Canada publishes several sets of productivity measures for the Canadian business sector and its major constituent subsectors (goods producing, services and the manufacturing subsectors) and industries. Each set of measures involves a comparison of the growth in output and input measures, but each relies on a different methodology. The concept of business sector excludes general government, private households, nonprofit organizations and the CSNA imputation of the rental value of owner-occupied dwellings. The business sector thereby excludes activities where it is difficult to draw inferences on productivity from the CSNA output measures. Such inferences would be questionable mainly because the CSNA output measures in these areas are based largely on incomes of inputs in constant prices, where productivity growth must therefore be zero by construction.

The traditional measure of labour productivity—output per hour—constitutes the first measure of productivity introduced by Statistics Canada in the early 1960s. Output, measured net of price change, is compared to labour input, measured as hours at work in the corresponding sector or industry.

The second set of measures covers multifactor productivity. In these measures, output is again measured net of price changes, but the input measure is an aggregate of hours worked and capital service flows. Multifactor productivity estimates have been developed in recognition of the role capital growth plays in output growth.

Both labour and multifactor productivity estimates have been published annually since 1961 and are updated on a yearly basis following the annual revisions made by the CSNA. Labour productivity estimates are *published* for 109 industries, compared with 101 for multifactor productivity as capital stock estimates are not always available at the same level of industry detail as the input-output tables.[5]

Statistics Canada's productivity estimates are based on a bottom-up approach to productivity measurement. Productivity indices are estimated with the most detailed data available by industry and by goods and services. Productivity indices are *computed* for 147 industries in the case of labour productivity and 122 industries in the case of multifactor productivity and then aggregated by steps up to the total business sector. This approach, which takes advantage of homogenous information available at a fine level of detail, proves to be superior to the aggregated approach as it significantly improves the quality of the measured aggregate productivity indices.[6]

A.3.1.1 Labour productivity and related measures

Labour productivity, calculated as the difference in the growth rate between GDP at basic price and the number of hours are available at the L-level of input-output tables (147 industries of the business sector). Appendix 2 provides a list of various levels of aggregation used by the productivity program. Since input-output tables are usually three years behind the reference year,[7] more current estimates are produced by using projections of GDP for a high level of aggregation—16 industries (the S-Level of input-output tables). These projections are based on a regression model developed by Mirotchie (1996), where the Fisher GDP is regressed on the Laspeyres GDP and a set of three time dummy variables capturing the lag between the reference year and the last year for which input-output tables are available.

Parallel to the labour productivity indices, Statistics Canada's productivity program also produces other performance indicators, such as indices of compensation per hour and unit labour cost. Indices of compensation per hour measure the hourly cost to employers of wages and salaries, as well as supplemental payments, which include employers' contributions to employment insurance taxes and payments for private health insurance and pension plans.

Unit labour costs measure the cost of labour input required to produce one unit of output. The index of unit labour costs is derived by dividing the compensation index in current dollars by the output index.

[5] Input-output tables, which constitute the major source of data used in the productivity estimates, provide information on input and output for 167 industries. See section A.3.2, "Estimation procedures and data sources."

[6] As stated by Jorgenson (1990), the assumptions that are necessary to admit the existence of an aggregate production function are rather heroic. Its existence requires that such a function be the same for all industries and that producers face identical prices. He showed that estimates of productivity made at the aggregate level under these assumptions may significantly depart from those obtained by aggregating detailed industry productivity estimates, based on less stringent assumptions.

[7] The reference year is the most current year for which annual series can be produced.

A.3.1.2 Multifactor productivity

The productivity program produces four categories of multifactor productivity indices, each of which responds to a different analytical need:

1. At the level of the business sector or its sub-sectors, multifactor productivity indices are measured as the value-added output per combined unit of labour and capital input.

2. At the industry level, comparisons of gross output (i.e., value-added *plus* intermediate inputs) with a broader set of inputs constitute a second category of multifactor productivity indices, known as the **industry** indices. They measure the growth in the gross output of an industry not accounted for by the growth in all of its inputs (capital, labour and the intermediate inputs, which are the materials and services purchased from other industries). These indices do not take into account the productivity gains that take place in the (upstream) industries that produce these intermediate inputs.

3. **Intra-industry** multifactor productivity indices, in which intra-industry sales are netted out from gross output, constitute a variant of the industry indices. In this instance, multifactor productivity growth is computed as if all establishments in a particular industry were integrated together into a single consolidated establishment covering the whole industry. That establishment sells all its output outside the industry and purchases all its intermediate inputs outside the industry. Accordingly, intra-industry purchases are excluded in the intra-industry integrated inputs.

4. None of the above multifactor productivity indices of a particular industry accounts for the productivity gains made by its upstream suppliers. By contrast, the **inter-industry** multifactor productivity indices do just that. They also include the productivity gains realized in the upstream industries supplying intermediate inputs.[8]

The inter-industry index measures the growth in the output of an industry not accounted for by the growth in all its primary inputs as well as by the growth in the primary inputs used in the production of its intermediate inputs by its direct and indirect industry suppliers. The inter-industry productivity indices take into account all the primary inputs that have been used in the business sector as a whole to produce a given bundle of goods and services. They may be seen as productivity indices attached to commodity bundles rather than to industries (Durand 1994).

These four measures clearly show that the concept of multifactor productivity can be defined for various industrial aggregation levels and also for various levels of vertical integration (measures 3 and 4) (see Figures 1 to 5). This variety of multifactor productivity indices are produced to satisfy various analytical needs expressed by data users. For example, in an effort to assess the performance of an economy as a whole in the production of some bundle of goods, it would be inappropriate to consider the declining industries with low productivity gains without also looking at the performance of the industries supplying them with goods and services. The ability of sellers of automobiles to pass on price savings due to productivity gains arises from productivity improvement not just in the auto assembly sector but also in auto parts, plastic, rubber, and a host of other upstream industries.

[8] The concept and the empirical estimates were first introduced by Cas and Rymes (1991). However, contrary to Cas and Rymes, the inter-industry multifactor productivity estimates produced by Statistics Canada include the capital stock in the primary inputs rather than in intermediate inputs.

Figure 1. Business Sector

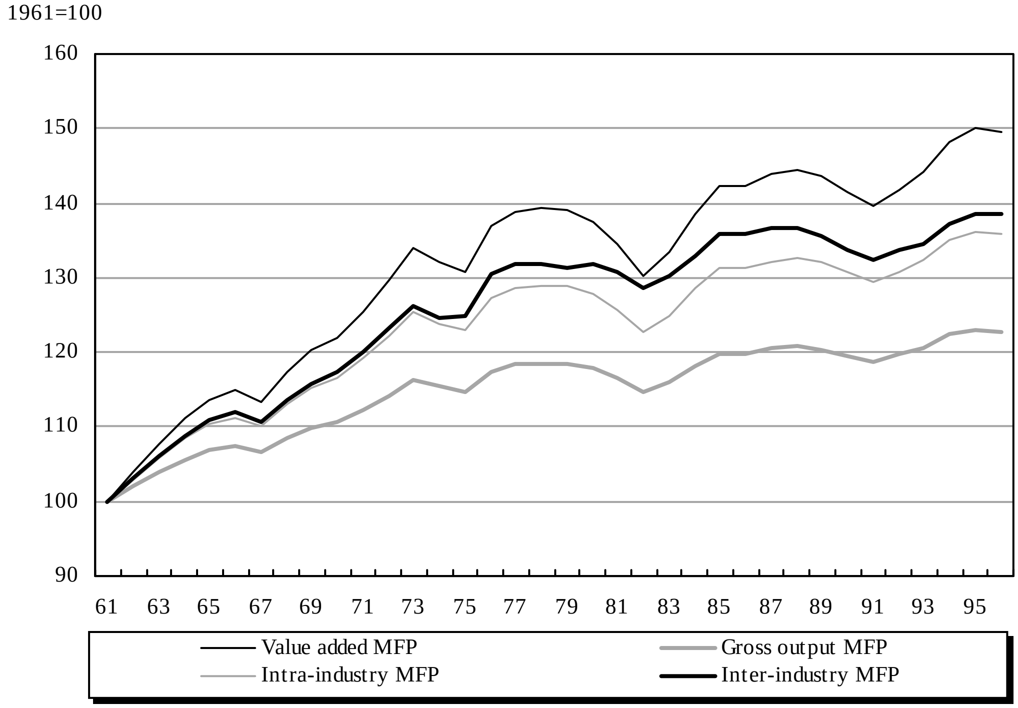


Figure 2. Business Sector excluding Agriculture

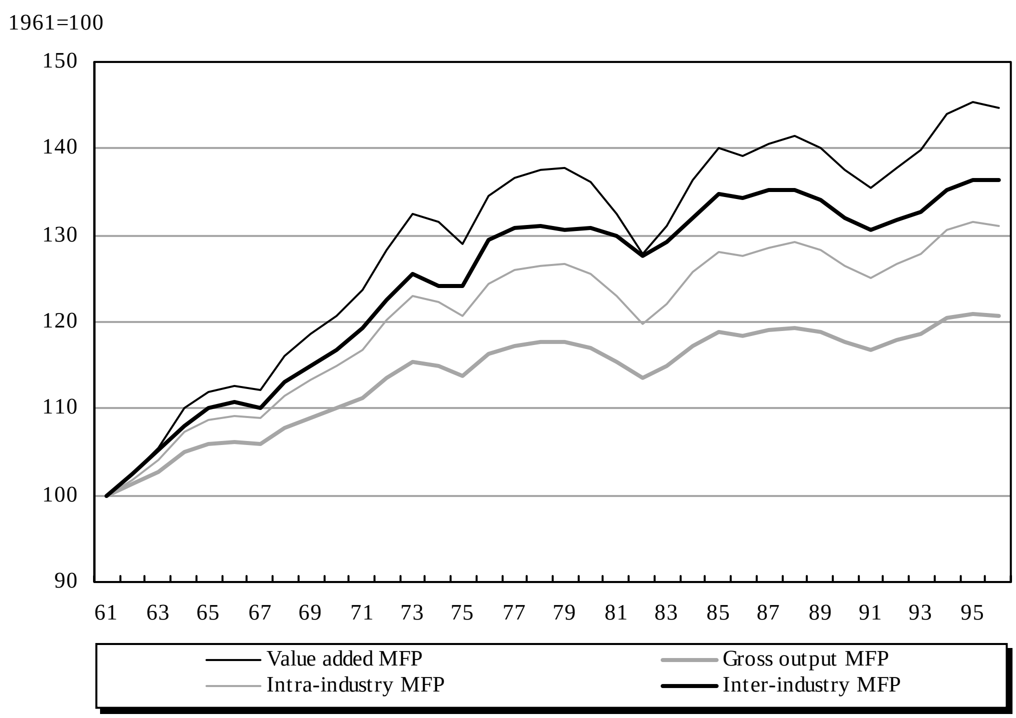


Figure 3. Business Sector – Goods Producing Industries

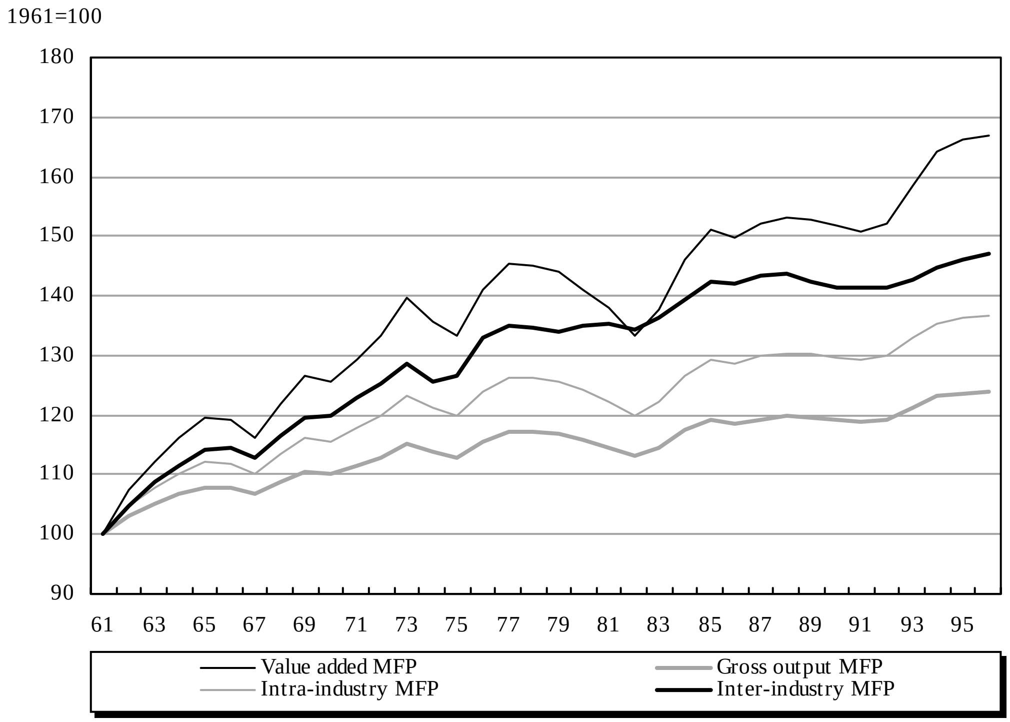


Figure 4. Business Sector – Services Producing Industries

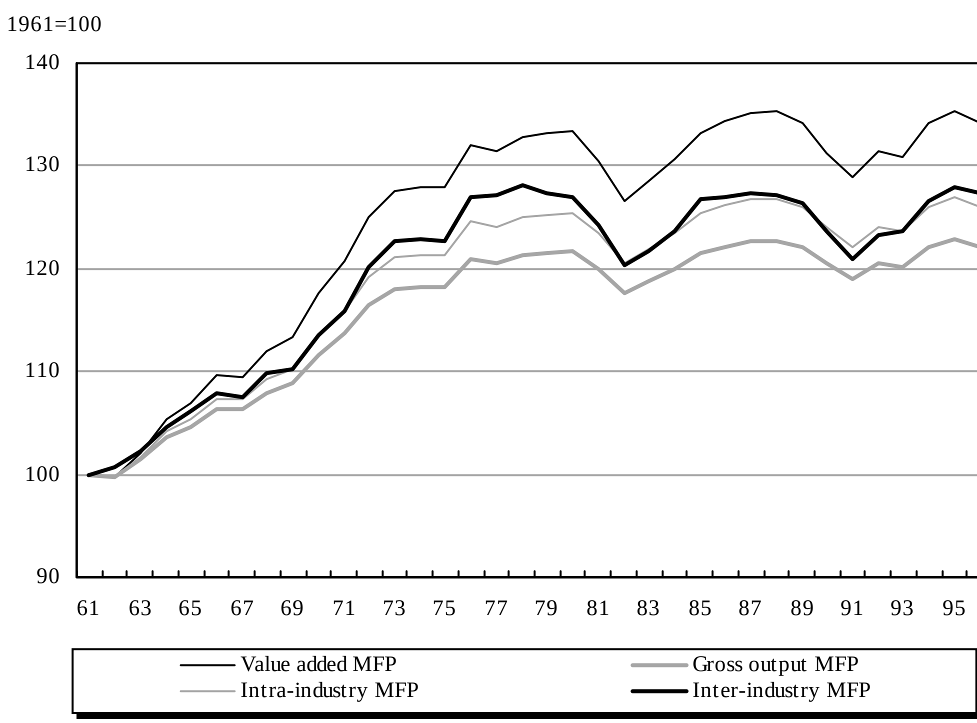
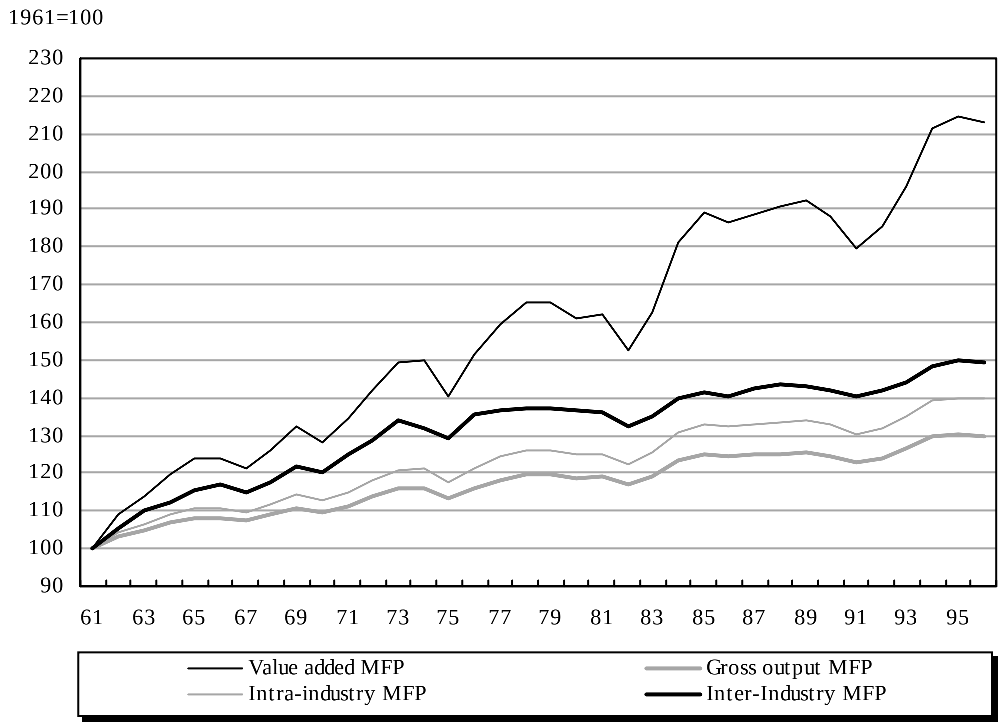


Figure 5. Manufacturing Industries



It is important to note that there are significant differences in the empirical estimates of different multifactor productivity measures (see Figures 1 to 5). The higher in the value added chain the estimate goes, the larger will be the productivity estimate. Comparisons that are made across countries that do not use the same level in the chain will contain inherent biases.

The relationship between the various multifactor productivity indices that are produced can be derived in a straightforward fashion.

The productivity growth estimates calculated using value added of an industry is just equal to the productivity growth estimates using gross output multiplied by an inflation factor, where that factor is equal to the industry's nominal gross output divided by its nominal value added. That is,

$$MFP_{VA} = \left(\frac{G}{VA}\right) \times MFP_G \tag{3}$$

where MFP_{VA} is multifactor productivity based on value added, MFP_G is multifactor productivity based on gross output, G is nominal gross output, and VA is nominal value added.

In the same way, intra-industry multifactor productivity using intra-industry value added is just

$$MFP_{II} = \left(\frac{G}{G_N}\right) \times MFP_G \tag{4}$$

where MFP_{II} is the intra-industry index, MFP_G is the gross output index, G is nominal gross output and G_N is the nominal gross output of an industry net of intra-industry sales.

Aggregating all industries together using the intra-industry measure of productivity is equivalent to considering all intermediate sales as intra-industry sales and leads to the elimination of all intermediate transactions in the business sector. This is equivalent to producing aggregate productivity measures based on value added. Because of vertical integration, the aggregate measure tends to be larger than the average of the industry measures. As a result, the higher the level of integration shown by the productivity measures, the higher the productivity gains (Durand 1996).

Like labour productivity, multifactor productivity estimates at a high level of industry detail are three years behind the reference year, but current information, based on a projection model, is available for the whole business sector and its major subsectors (Mirotchie 1996). For the multifactor productivity estimates, the model projects current information on the Fisher indices of GDP, capital stock and hours, on the basis of the Laspeyres indices of these variables and dummy trend variables.

A.3.1.3 Availability of results

New results on labour productivity (and related measures) and multifactor productivity (compensation per hour and unit labour cost) announced in Statistics Canada's official news release, *The Daily,* are published twice a year. These estimates are highly current for major sub-sectors of the business sector (one year behind the reference year) but they are three years behind the reference at the industry level. A limited amount of the most current data is provided in the news releases, but the historical series can be accessed from Statistics Canada's CANSIM database or from its Web site at www.statcan.ca. A list of CANSIM matrices can be found in Appendix 4.

Preliminary estimates of labour productivity indices and related measures (unit labour cost and compensation per hour) are generally announced in late April each year (every June for the multifactor productivity estimates). The revisions to the labour productivity estimates (and their related measures), along with the production of more current information at the industry level, are published in November (December for revised multifactor productivity estimates) of the same year, following the release of the input-output tables' results.

A.3.2 Estimation procedures and data sources

A.3.2.1 General overview

In order to produce productivity growth estimates, various data sources from Statistics Canada's survey areas and the System of National Accounts are integrated. In particular, the productivity program requires data from the following:

1. the Input Output Division, which provides the structure of the economy (in terms of industries, the commodities produced and used, and how they change over time) in both current and constant prices that is so essential to the production of aggregate estimates that are built from the ground up at the industry level;

2. the Labour Statistics Division, which provides employment numbers and hours worked to estimate the labour input;

3. the Investment and Capital Stock Division, which provides estimates of year-end net capital stock to estimate capital input; and

4. the Industry Measures and Analysis Division, which produces current estimates of GDP in constant prices, for preliminary estimates of productivity for the three most recent years.

Data that come from these different sources are conceptually adjusted and reconciled for accuracy and consistency. As such, the production of productivity measures serves as an important source of quality control on the various data series that are used in the productivity program. In almost all cases, the data received are transformed into a form that is appropriate for the calculation of productivity estimates. Using the raw data would be inappropriate, or at least would provide productivity estimates that are not as precise as required.

Efforts are made to integrate the data to ensure that measures of outputs and inputs cover the same sectors. For example, industry coverage of the productivity measures includes tenant-occupied housing but does not cover owner-occupied dwellings. Published measures of capital stock do not distinguish between these two activities. Therefore, measures of capital for tenant-occupied housing are derived for the purposes of productivity estimation.

The input-output tables are used to take into account changes in the industrial structure in the weighting procedures that calculate rates of change of outputs and inputs. Calculated rates of change in inputs or outputs are sensitive to the weights that are used to aggregate the 469 commodities that make up outputs or inputs. If these weights are not calculated correctly, estimates of rates of change will be incorrect. Using the input-output tables, the methodology in place allows these weights to change each year (using a Fisher chain weight) so as to keep the industrial structure up to date—both in the calculation of changes in inputs and changes in outputs.[9]

A.3.2.2 Output and input data: Transformation and integration

Statistics Canada's productivity measures are closely linked to the input-output tables. The input-output tables, along with data on hours and capital stock in constant prices, are used to produce the various measures of productivity growth. The production of the annual productivity estimates requires several transformations to the raw data. These transformations involve: a) the choice of the level of aggregation; b) the selection of business sector industries; c) the decision on the valuation of the outputs and inputs; and d) the assumptions on the compensation of primary inputs. Once these transformations are implemented, the resulting data on input and output are integrated with hours and capital stock data.

Transformation of data

Level of detail: Annual input-output data are imported from the input-output tables at the L-level (link) of aggregation from 1961 to the most recent year (usually three years behind the reference year) and include both business and non-business industries. This is the most detailed level for which there is a consistent definition of industries and commodities across all years. All in all, the make (output) and use (intermediate inputs)

[9] Input-output tables in constant prices make use of Laspeyres indices of quantities chained every five years.

matrices of input-output tables have 167 industries (147 non-dummy business industries and 7 dummy business industries for a total of 154 business industries and 13 non-business industries) and 469 commodities excluding indirect taxes and subsidies and compensation of the primary inputs. Indirect taxes and subsidies by commodity and by industry are compiled separately from the intermediate inputs to which they apply.

Compensation of primary inputs includes the following items applicable to incorporated businesses operating in all industries: wages and salaries and supplementary income for the compensation of labour, and other operating surplus for the compensation of capital. Mixed income includes the compensation of labour and capital employed by the unincorporated portion of the business sector.

Coverage of the business sector: Since productivity cannot be measured for non-business industries (general government, private households, non-profit organisations and owner-occupied dwellings) these industries are excluded from both the *make* and the *use* matrices in current and constant prices.[10] The same holds true for dummy industries that are fictitious industries in the input-output tables created to route real commodity consumption to other industries via dummy commodities.

In principle, dummy industries have to be excluded since they have no primary inputs and have intermediate inputs that grow at the same rate as their output, which leave them with zero productivity gains. The exclusion rules are the same as those applied to non-business industries. Therefore, only the 147 non-dummy business industries are retained in the production of productivity estimates.

The owner-occupied portion of residential housing classified in the Finance, Insurance and Real Estate subsector is excluded from the coverage of the business sector for two reasons: a) there is no adequate accounting of labour input of this industry and b) since the U.S. Bureau of Labor Statistics does not account for this industry for the same reasons, it allows Statistics Canada to construct comparable productivity estimates to those of the United States.

Valuation base for outputs, inputs and compensation: All input and output data are adjusted to correspond to prices effectively received from the sale of output and the prices paid as a result of the purchase of inputs. This means that the value of inputs should include taxes and exclude subsidies. Similarly, the value of output is taken net of output taxes and subsidies. To effect this, the value of commodity indirect taxes is distributed over the input and output commodities to which they apply. Subsidies are similarly allocated to the inputs and outputs to which they apply. This means that the concept of GDP used in the productivity estimates is not the same as the one produced by the input-output tables. GDP from the input-output tables is at factor cost, whereas GDP from the productivity program is at basic prices (i.e., GDP at factor cost *plus* indirect taxes on production *minus* subsidies on production).

The following three classes of indirect taxes are considered in the valuation of the inputs: indirect taxes on products, import duties, and indirect taxes on production. The former two apply to the intermediate inputs and the latter applies to the capital compensation. Import duties are included in the import prices of commodities and enter into the intermediate input prices valued at purchaser's prices. The indirect taxes on products

[10] The *make (use)* matrix is a matrix of the input-output tables that reflects the commodities produced (used) by the different industries.

are included in the purchaser's valuation of intermediate commodity input prices. The indirect taxes on production include property taxes as a major component and are considered part of the capital compensation.

Capital income is measured gross of direct income taxes and other non-commodity indirect taxes (mostly property taxes). Similarly, labour income is gross of income taxes.

Compensation of primary inputs: The compensation of the primary inputs in the input-output tables consists of the following variables: a) wages and salaries, b) supplementary labour income, c) mixed income, d) other operating surplus and e) net indirect taxes on production. Wages and salaries and supplementary labour income measure the compensation of paid workers. Other operating surplus is the gross capital income of incorporated businesses and includes profits before taxes, corporate income taxes, depreciation and rents on natural resources. It is computed residually in the input-output accounts as total income minus all other input costs. Net indirect taxes on production include mostly property taxes and are included in the measure of capital income.

Mixed income constitutes the earnings for both capital and labour inputs arising from the unincorporated portion of the business sector and is taken from tax records. Therefore, it includes the labour income of the self-employed and unpaid family workers, both of which are constructed by the productivity program.

The value of labour services of self-employed persons is an imputed value. The imputation is based on the assumption that the value of an hour worked by a self-employed person is the same as the value of an hour worked by an average paid worker in the same industry. This assumption is based on the premise that labour services are contracted on a temporal basis, and a measure of labour compensation should not reflect returns on investment or risk taking. However, an adjustment is made in the case of self-employed persons such as doctors, dentists, lawyers, accountants and engineers. In these cases, the average earnings of paid workers in the same industry tend to be lower than the earnings of the self-employed workers. Although self-employed workers are in the majority in these industries, the imputation of earnings for these workers at the average rate of the paid workers in these industries tends to underestimate the income of the self-employed. In this case, direct evidence on average labour income of these workers is used. Finally, for a given industry, when the imputed income for self-employment produces a higher result than total mixed income, the imputed value is made equal to mixed income.

Unpaid family workers, while not directly compensated for their services, are not a free resource, and their contribution is reflected in the net income of the firm where they are employed. However, no labour income is imputed to unpaid family workers.[11] There is no valid basis for measuring the value of their services, and it is judged that less error is generated by their exclusion from measures of labour compensation than by imputing labour income to them at the same rate as paid workers. The number of unpaid family workers is insignificant in most industries.

Labour income of self-employed and unpaid family workers is then subtracted from mixed income to arrive at the concept of other capital income, a measure of capital compensation of unincorporated businesses used by the productivity program. Other capital income is then aggregated with other operating surplus and net indirect taxes on production to obtain the total capital compensation of incorporated and unincorporated businesses.

[11] Nevertheless, data on hours and employment are available for unpaid family workers.

Integration of hours and capital stock to the transformed input-output tables

The input-output tables in constant prices do not contain data on hours worked and the end-year net capital stock in constant prices. These data undergo several conceptual transformations within the productivity program prior to their integration into the transformed input-output tables.

Labour input: The measurement of labour input requires several refinements to the concept of the head count of employees, the simplest and least differentiated measure of labour input. Such a measure neither recognizes changes in the average work time per employee nor does it reflect the role of self-employed or even differences in labour quality.

The measure of labour input starts with the concept of total jobs, consisting of wage and salary earners, self-employed and unpaid family workers, and then converts units from simple job counts to total hours worked. The rapid increase in non-standard types of employment (part-time, self-employment, etc.) stresses the importance of using hours worked as the unit of labour input in productivity measurement because they bear a closer relationship to the concept of labour services than simple job counts.

The number of hours worked may not be identical to the number of hours paid, mainly as a result of holidays and paid annual sick leave. Hours worked, rather than hours paid, is used to estimate the labour input measure because it is more closely linked to the production process.

At present, estimates of labour input used by Statistics Canada's productivity program implicitly account for differences in the composition of the labour force by industry (quality). Statistics Canada simply aggregates different types of labour at the industry level to produce an industry total. But the growth of the labour input at the level of the business sector and its constituent subsectors is the weighted sum of the number of hours worked by industry where the weights are defined in terms of the industry's share in the total labour compensation. These shares or weights will be comparatively large for industries with above-average wages and relatively small for industries with below-average wages. Assuming that above-average wages reflect above-average skills of the work force, higher weights will be applied to the growth rates of industries with a higher quality of labour. As relative wages increase in an industry, the weights will increase.

Capital input: Capital stock estimates are constructed by using the perpetual inventory method, where successive net capital stock in constant prices is related by the following equation:

$$K_t = I_t + (1 - \delta) K_{t-1}, \tag{5}$$

where K_t is the real capital stock at time t, I_t is the real investment, and δ is the (constant) rate of depreciation of the capital stock; δ need not be a constant, but almost always is assumed to be. To construct a capital stock series, one usually starts at an initial period 0 with a measure of the initial capital stock, K_0, and then calculate successive values of K_t by substituting the depreciation rate and the elements of an investment series into (5). By successive backward substitution for K_{t-1} in (5), one can relate K_t directly to the initial value for the capital stock K_0. K_t becomes a weighted sum of all past levels of investment and the depreciated value of the initial real capital stock

$$K_t = \sum_{i=0}^{t-1} (1-\delta)^i I_{t-i} + (1-\delta)^t K_0. \tag{6}$$

The amount of capital produced from a given stream of investment depends on the depreciation profile that is used. Less capital is produced when the depreciation profiles are relatively steep—where the percentage of value lost in the early years of an asset's life is large.[12] Statistics Canada produces three estimates of capital stock based on three alternative depreciation profiles: the geometric, the delayed and the linear (Statistics Canada 1999). These are derived from:

$$F(\tau, L) = \begin{cases} \delta(1-\delta)^{(\tau-1)} & (\text{geometric}) \\ \frac{L-(\tau-1)}{L-\beta(\tau-1)} - \frac{L-\tau}{L-\beta\tau} & (\text{delayed}) \\ \frac{1}{L} & (\text{linear}) \end{cases} \tag{7}$$

where F represents the value of $1 of original investment at age τ and L is the length of life. The geometric distribution assumes that the rate of depreciation is a constant. In the geometric function, δ is set equal to $\frac{R}{L}$, where R is an arbitrary constant $(= 2)$ and $L(> 2)$ the length of life; in the delayed function, β is the curvature parameter which takes the value 0.75 for structures and 0.5 for machinery and equipment. At present, the geometric method is normalized so that the full value of an asset depreciates over its life rather than over an indefinite time span (this is the truncated geometric method).

In addition, the productivity program undertakes several changes to the estimates of capital stock net of geometric depreciation to arrive at a measurement of capital stock that is consistent with the concept of the business sector. The business sector is made up of the private non-residential and the residential components.

Private non-residential capital stock: The following 1980 Standard Industrial Classification for establishments (SIC-E) industries are deleted from the private and public estimates of capital stock published by the Investment and Capital Stock Division to arrive at the private non-residential estimates of capital stock:

- N8100 (Federal Government Service Industries)
- N8200 (Provincial and Territorial Government Service Industries)
- N8300 (Local Government Service Industries)
- O8510 (Elementary and Secondary Education)
- O8520 (Post-Secondary Non-University Education)

[12] While different assumptions about depreciation have a large effect on the level of capital stock, they have much less of an effect on the rate of growth of the capital stock. See Chapter 3.

- O8530 (University Education)
- P8610 (Hospitals)

Residential capital stock: Data on total residential capital stock cover both the tenant-occupied and owner-occupied portions of the economy. Only the former is part of the business sector covered by the productivity program. The breakdown of total residential capital stock between tenant-occupied and owner-occupied portions is made on the basis of gross rent obtained from the input-output tables. The rented portion of the residential sector is then added to the non-residential capital stock to arrive at the business sector's capital stock.

In order to produce capital stock for each industry, capital can be created by simply summing across all asset categories or by deriving a weighted sum using the relative shares of each category in total compensation, where the latter are derived using rental rates of capital. At present time, Statistics Canada uses a simple aggregate across three asset classes (machinery and equipment, buildings, and engineering construction). However, in aggregating capital stock across industries, it weights each industry by its return on capital as described above. Industries with higher cost of capital will implicitly receive a higher weight using this methodology, and changes in relative cost of capital will be reflected in changing weights.

A.3.2.3 On Quality Adjustments

The measurement of multifactor productivity requires estimates of increases in factor inputs. As noted previously, Statistics Canada does so with a measure of hours-worked for labour inputs and real dollars of capital stock on the capital stock. Others (Jorgenson and Griliches, 1967; Jorgenson, 1990) have suggested that adjustments be made to the quality of each of these inputs. For example, this alternate methodology divides hours-worked into various categories (for example, males as opposed to females) and the rates of growth of each are weighted by the relative share of total wages going to each. This procedure gives higher weights to the growth rates of the group earning higher wages—and implicity assumes that higher wages are representative of higher marginal productivity and of higher quality.

This procedure redistributes some of the growth in the multifactor productivity reported here to labour and capital. If multifactor productivity is meant to help us understand the sources of growth, this procedure adds to our information in this regard. For output growth can now be attributed not just to increasing labour but to increasing labour of a certain type. As such this exercice serves to usefully supplement our existing measures and Statistics Canada is working on providing such estimates as a supplement to its normal program.

But it should be noted that these estimates are not without problems. In the first place, differences in wages may not just reflect differences in marginal products. For example, some would argue that male/female wage differentials partially reflect discrimination in labour markets. Ascribing all gender wage differentials to quality differences may be unjustified. And deciding just how much of the differentials to ascribe to real quality differences is not an easy or very precise task.

Second, this approach gradually reduces the residual that multifactor productivity is measuring towards zero—and as such the measure becomes less useful as a measure of technical progress that many users of the data use it for. Nor should we expect the quality corrected measure to be as closely related to measures of industry performance.

Finally, quality adjusted multifactor productivity series would probably have even greater measurement problems than are outlined in Chapter 3.

Despite these shortcomings, Statistics Canada is working on providing new supplementary measures in this area that will be released some time next year.

To construct the growth rate of capital stock, the productivity program makes use of the following sources of information:

1. The private non-residential and residential estimates of capital stock net of geometric depreciation in constant prices produced by the Investment and Capital Stock Division;
2. The information on compensation of capital constructed by the productivity program from the input-output tables.

A.3.2.4 On the 1997 historical revision of the System of National Accounts

Both labour and multifactor productivity measures use data that are periodically revised. About once every five years, the CSNA is rebased to keep up with the evolution of prices in the economy (Jackson 1996). In other words, the constant-price aggregates are recalculated in terms of the prices of a more recent time period. In addition, the System is revamped about once a decade to introduce new accounting conventions and improved methods of estimation. The recent changes to the System also reflect the need to bring the CSNA in line with the 1993 United Nations System of National Accounts (SNA), recommendations that will improve international comparability.[13]

The choice of a base year for the constant price estimates of output and capital stock is arbitrary, but nevertheless important. The *level* of output and capital stock and their components for any particular year can be quite different if the base year is altered. The last rebasing coincided with the release of the GDP estimates for the first quarter of 1996. At that time, the constant price series were shifted from 1986 to 1992 price weights. When the series are recalculated in this manner, the new weights are normally applied from the new base period forward. The estimates for previous years are not normally recalculated using the relative prices of the new base year in the CSNA. Rather, the already calculated constant price estimates for previous years are mechanically linked, or scaled, so as to join up with the new series. Each 'component' series is linked independently and, in some cases, the results are forced to add up through the introduction of 'adjusting entries' series (Statistics Canada 1975: 279). In this way, the growth patterns for earlier years are preserved.

Adjusting entries are calculated for GDP and its subcomponents, like gross capital formation by the CSNA. However, no adjusting entries are presently calculated to estimate capital stock and gross capital formation by the Investment and Capital Stock Division, so that their rebasing changes the growth rate of the capital stock series before the new base year. For this reason, publicly available real GDP and real capital stock estimates are not compatible. The productivity program, however, uses data from these sources that are compatible. The productivity program also uses a chained-type Fisher index in its measure of real output, labour input and capital input to address the problem that arises when rebasing is done periodically.[14] This index is a geometric mean of the chained-weighted Laspeyres and Paasche indices. Changes in this measure are calculated using the weight of adjacent years. These annual changes are 'chained' (multiplied) to form a time series that allows for the continuous incorporation of the effect of changes in relative prices and in the composition of the series over time.

[13] For a comprehensive review of the 1997 historical revision of the CSNA, see Lal (1998).
[14] Before the 1997 historical revision, the program used the Törnqvist chain index.

The 1997 historical revision also made some changes to the previous treatment of several industries in the input–output tables. The main change is the disappearance of the Government Royalties on Natural Resources Industry. In the revised version of the tables, this industry no longer exists and the commodity having the same name is now grouped with other operating surplus (capital income).

A.4 Calculation procedures

A.4.1 Labour productivity

The labour productivity (LP), or output per hour, index between two adjacent years t and $t-1$, is computed as a real value-added Fisher index[15] $\left(Y^F_{i,t/t-1}\right)$ of industry $i\left(i=1,2,\ldots,I\right)$ divided by an index of hours worked in that industry $\left(H_{i,t/t-1}\right)$. At the business sector level, we have

$$LP_{i,t/t-1} = Y^F_{i,t/t-1} \div H_{i,t/t-1}. \quad (8)$$

The Fisher index of real value added is computed at the industry level i based on information on prices and quantities of various commodities j produced by this industry. This is accomplished in several steps:

First, the Laspeyres $\left(Y^L_{i,t/t-1}\right)$ and Paasche $\left(Y^P_{i,t/t-1}\right)$ indices of real value added $Y_{i,t/t-1}$, for t and $t-1$ consecutive periods so as to form chain indices, are computed respectively as[16]

$$Y^L_{i,t/t-1} = \sum_{j=1}^{469}\left(\frac{Y_{i,j,t}}{Y_{i,j,t-1}}\right)\cdot\left(\frac{p_{i,j,t-1}\cdot Y_{i,j,t-1}}{\sum_{j=1}^{469} p_{i,j,t-1}\cdot Y_{i,j,t-1}}\right), \quad (9)$$

and[17]

$$Y^P_{i,t/t-1} = \sum_{j=1}^{469}\left(\frac{Y_{i,j,t}}{Y_{i,j,t-1}}\right)\cdot\left(\frac{p_{i,j,t}\cdot Y_{i,j,t-1}}{\sum_{j=1}^{469} p_{i,j,t}\cdot Y_{i,j,t-1}}\right). \quad (10)$$

[15] Defined as the geometric mean of the Laspeyres and Paasche chain indices.

[16] Recall that real value added is computed as real gross output net of real intermediate inputs.

[17] Or alternatively $Y^P_{i,t/t-1} = \left[\sum_{j=1}^{469}\left(\frac{Y_{i,j,t-1}}{Y_{i,j,t}}\right)\cdot\left(\frac{p_{i,j,t}\cdot Y_{i,j,t}}{\sum_{j=1}^{469} p_{i,j,t}\cdot Y_{i,j,t}}\right)\right]^{-1}$.

Second, the Fisher chain index, $Y^F_{i,t/t-1}$, is calculated as

$$Y^F_{i,t/t-1} = \sqrt{Y^L_{i,t/t-1} \times Y^P_{i,t/t-1}}. \tag{11}$$

The Fisher index of real value-added is then constructed at a higher level of industrial aggregation (e.g., the manufacturing sector):

$$Y^F_{t/t-1} = \sum_{i=1} \omega_{it} \cdot Y^F_{i,t/t-1}. \tag{12}$$

where $\omega_{it} = \frac{V_{it} - M_{it}}{\left(\sum_{i=1} V_{it} - M_{it}\right)}$ represents the share, in terms of nominal value-added (where V_{it} and M_{it} are, respectively, gross output and intermediate inputs), of the industry i in year t.

The index of hours worked is computed as

$$H_{i,t/t-1} = \frac{\sum_{i=1}^{I} H_{i,t}}{\sum_{i=1}^{I} H_{i,t-1}}. \tag{13}$$

The computation of labour compensation per hour worked parallels the computation of output per hour.

Unit labour costs (ULC), computed as labour compensation (LC) per unit of output, highlights the relationships between unit labour costs, hourly compensations and labour productivity:

$$ULC_{i,t} \equiv \left(\frac{LC_{i,t}}{Y_{i,t}}\right) = \left(\frac{LC_{i,t}}{H_{i,t}}\right) \div \left(\frac{Y_{i,t}}{H_{i,t}}\right). \tag{14}$$

Unit labour cost is identically equal to the ratio of average hourly compensation to labour productivity; thus, unit labour costs will increase when average hourly compensation grows more rapidly than labour productivity.

A.4.2 Multifactor productivity

Like the labour productivity estimates, multifactor productivity estimates make use of a superlative aggregation scheme based on the Fisher chained index on both outputs and inputs across commodities and industries.

Estimates of the Fisher chained index require estimates of prices and quantities at a high level of detail, which is the commodity (j) for both gross output $\left(Q_{ij}\right)$ and intermediate inputs $\left(M_{ij}\right)$, and the industry (i) for capital $\left(K_i\right)$ and hours $\left(H_i\right)$.

The following steps are followed during the construction of the Fisher index for these variables.

A.4.2.1 Output and intermediate inputs

Let p_{ijt} be the price of commodity j produced by the industry i in year t and w_{ijt} the price of the intermediate input j used by the industry i during in year t, whereas Q_{ijt} and M_{ijt} represent their corresponding quantities.

- The Fisher index of output is computed at the industry level i based on information on prices and quantities of various commodities produced or used by this industry. First, the Laspeyres $\left(Q^L_{i,t/t-1}\right)$ and Paasche $\left(Q^P_{i,t/t-1}\right)$ indices of output Q_{it}, for t and $t-1$ consecutive periods are computed respectively as

$$Q^L_{i,t/t-1} = \sum_{j=1}^{469}\left(\frac{Q_{i,j,t}}{Q_{i,j,t-1}}\right)\cdot\left(\frac{p_{i,j,t-1}\cdot Q_{i,j,t-1}}{\sum_{j=1}^{469} p_{i,j,t-1}\cdot Q_{i,j,t-1}}\right), \tag{15}$$

and

$$Q^{\mathcal{P}}_{i,t/t-1} = \sum_{j=1}^{469}\left(\frac{Q_{i,j,t}}{Q_{i,j,t-1}}\right)\cdot\left(\frac{p_{i,j,t}\cdot Q_{i,j,t-1}}{\sum_{j=1}^{469} p_{i,j,t}\cdot Q_{i,j,t-1}}\right). \tag{16}$$

Second, the Fisher index, $Q^F_{i,t/t-1}$, is calculated as

$$Q^F_{i,t/t-1} = \sqrt{Q^L_{i,t/t-1}\times Q^P_{i,t/t-1}}. \tag{17}$$

- The Fisher index of output, $Q^F_{t/t-1}$, is then constructed at a higher level of industrial aggregation (e.g., the manufacturing sector)

$$Q^F_{t/t-1} = \sum_{i=1}\omega_{it}\cdot Q^F_{i,t/t-1}, \tag{18}$$

where $\omega_{it} = \frac{V_{it}}{\sum_{i=1}^{146} V_{it}}$ represents the share in terms of gross output in nominal prices V_{it} of industry i in year t.[18]

[18] The same approach is developed for the multifactor productivity estimates based on the concept of value added.

A.4.2.2 Capital stock and hours

Much like the estimates of output and intermediate inputs, estimates of the Fisher chain index of capital input and labour input require series on prices and quantities. Series on quantities of labour and capital of industry i in year t are defined in terms of the number of hours h_{it} and the stock of capital in constant prices net of geometric depreciation k_{it}. The price series are constructed implicitly using the ratio of labour compensation W_{it} to the number of hours h_{it} for labour, and the ratio of capital compensation R_{it} (see "Compensation of primary inputs" in section A.3.2.2) to capital stock k_{it}, that is

$$r_{it} = \frac{R_{it}}{k_{it}}, \tag{19}$$

and

$$v_{it} = \frac{W_{it}}{h_{it}}, \tag{20}$$

where r_{it} and v_{it} represent, respectively, the (average) return on capital per unit of capital and the (average) hourly labour compensation. The construction of the Fisher chain index of capital input, $\left(K^F_{i,t/t-1}\right)$, at the industry level proceeds as follows (and similarly for labour):

- First, the Laspeyres $\left(K^L_{i,t/t-1}\right)$ and Paasche $\left(K^P_{i,t/t-1}\right)$ indices of capital input are computed as

$$K^L_{i,t/t-1} = \frac{k_{i,t} \cdot r_{i,t-1}}{k_{i,t-1} \cdot r_{i,t-1}} \,;\, K^P_{i,t/t-1} = \frac{k_{i,t} \cdot r_{i,t}}{k_{i,t-1} \cdot r_{i,t}} \tag{21}$$

The Fisher index of the capital input is then calculated as

$$K^F_{i,t/t-1} = \sqrt{K^L_{i,t/t-1} \times K^P_{i,t/t-1}}. \tag{22}$$

- The Fisher index of capital input, $\left(K^F_{t/t-1}\right)$, at a higher level of aggregation (e.g., manufacturing sector) is calculated as

$$K^F_{t/t-1} = \sum_{i=1}^{122} \omega_{it} \cdot K^F_{i,t/t-1} \tag{23}$$

where $\omega_{it} = \frac{R_{it}}{\sum_{i=1}^{122} R_{it}}$ represents the capital compensation share of the current year t of the industry i in the whole business sector.

The weight ω_{it} for each industry is based on the share of the compensation of each of the primary inputs, which makes the construction of capital input and labour input used for the multifactor productivity indices similar, albeit not identical. In that sense, a partial adjustment for the quality of the primary inputs is obtained as the change in each of these inputs used by an industry is aggregated to the economy-wide level using each industry's share in total compensation as aggregation weights. The capital (labour) weight will be large for industries displaying an above average internal return of capital (labour compensation) and small for those that do not. The weights will increase for those industries whose relative return (wage) increases over time. Some of the change in quality of capital (labour) would then be accounted for, assuming that above-average internal return of capital (labour compensation) reflect above-average 'performance' of capital (labour).

A.4.2.3 Aggregation of the inputs

- The Fisher index of the aggregate input $\left(I^F_{t/t-1}\right)$ is calculated as follows:

$$I^F_{t/t-1} = \overline{s}^K_{t/t-1} \times K^F_{t/t-1} + \overline{s}^L_{t/t-1} \times L^F_{t/t-1} \ , \qquad (24)$$

where $\overline{s}^L_{t,t-1} = \frac{1}{2}\left(s^L_t + s^L_{t-1}\right)$, $\overline{s}^K_{t/t-1} = 1 - \overline{s}^L_{t/t-1}$ and $s^\iota_{t/t-1}$ represents the share of the input $\iota\,(\iota = K, L)$ (in terms of its compensation) in the value of output (assumed to be measured in terms of value added).[19]

- The growth rate of the multifactor productivity index $MFP^F_{t/t-1}$ captures the proportional change over time of technical progress (Δ refers to discrete changes in percentage with respect to time):

$$\begin{aligned} \Delta MFP^F_{t/t-1} &= \Delta Q^F_{t/t-1} - \Delta I^F_{t/t-1} \\ &= \Delta Q^F_{t/t-1} - \left(\overline{s}^K_{t/t-1} \times \Delta K^F_{t/t-1} + \overline{s}^L_{t/t-1} \times \Delta L^F_{t/t-1}\right) \ , \end{aligned} \qquad (25)$$

where $Q^F_{t/t-1}$, $K^F_{t/t-1}$ and $L^F_{t/t-1}$ are the Fisher-Ideal indices of output, capital and labour, respectively. In other words, multifactor productivity is simply the growth in output minus the output-share-weighted growth in inputs.

19 $s^L_t = \frac{\text{Labour compensation}}{\text{nominal output}}$ and s^K_t is obtained residually as a result of the constant returns to scale assumption $s^K_t + s^L_t = 1$.

A.4.3 Labour productivity, multifactor productivity and technology

This part develops the basic algebra of productivity accounting and then relates multifactor productivity measures to single-factor (say labour) productivity indices.

Rewrite $\Delta MFP^F_{t/t-1}$ as $\left(\bar{s}^K_{t,t-1} + \bar{s}^L_{t,t-1}\right) \times \Delta MFP^F_{t/t-1}$ and collect terms in (25).[20] This yields:

$$\Delta MFP^F_{t/t-1} = \bar{s}^K_{t/t-1}\left(\Delta Q^F_{t/t-1} - \Delta K^F_{t/t-1}\right) - \bar{s}^K_{t/t-1}\left(\Delta Q^F_{t/t-1} - \Delta L^F_{t/t-1}\right). \quad (26)$$

Equation (26) has a straightforward interpretation, since the terms between parentheses represent, respectively, the rate of growth of capital productivity and labour productivity. Equation (26) indicates that multifactor productivity is a weighted average of capital productivity and labour productivity, where the weights are respectively output shares of capital and labour. When capital and labour productivity grow at the same rate, because of Hicks neutral technical change, multifactor productivity $\Delta MFP^F_{t/t-1}$ is simply the common rate of capital and labour productivity growth.

To provide an interpretation of elements affecting labour productivity, subtract $L^F_{t/t-1}$ from the left-hand side and $\left(\bar{s}^K_{t/t-1} + \bar{s}^L_{t/t-1}\right) \times \Delta L^F_{t/t-1}$ from the right-hand side of (25), and then collect terms. This yields:

$$\left(\Delta Q^F_{t/t-1} - \Delta L^F_{t/t-1}\right) = \Delta MFP^F_{t/t-1} + \bar{s}^K_{t/t-1}\left(\Delta K^F_{t/t-1} - \Delta L^F_{t/t-1}\right), \quad (27)$$

which is interpreted as follows. The growth in labour productivity is the sum of two terms: the effects of technological progress $\Delta MFP^F_{t/t-1}$ and the capital-share-weighted change in the capital-to-labour ratio. Rapid gains in labour productivity in the 1960s, for example, were attributable partly to neutral technological progress, but also due to the fact that capital per worker increased substantially, i.e. $\Delta K^F_{t/t-1} - \Delta L^F_{t/t-1} > 0$. Hence, rapid investment in plant and equipment leads to increases in labour productivity.

Note that this growth accounting framework does not explain why $\Delta K^F_{t/t-1} - \Delta L^F_{t/t-1}$ was positive; that is a different issue. What (27) reveals is that measured labour productivity is positively related to growth in the capital-to-labour ratio and vice versa.

[20] Recall that $\bar{s}^K_{t,t-1} + \bar{s}^L_{t,t-1} = 1$.

A.5 International comparisons of productivity growth

A.5.1 Introduction

Since its inception, Statistics Canada's productivity program has established the international comparison of productivity performance as one of its priorities.[21] Attempts over the years to improve the comparability between Canada's productivity measures to those of its major trading partners have been undertaken mainly because comparisons provide information on the competitive position of Canada in foreign trade, which has an important influence on the Canadian economy and employment.

Because statistical concepts and methods vary from country to country, international comparisons of statistical data can be misleading. Differences in sources, concepts and methods used in preparing productivity estimates often lead to substantially different results. This is rightfully worrisome for many users who would like to know which ones they should use in their analysis of current economic conditions.

This section deals with the comparability of productivity estimates from various sources with special emphasis on the estimates produced by the OECD, the U.S. Bureau of Labor Statistics and Statistics Canada. The purpose of this section is not so much to suggest the best estimates but merely to emphasize the differences underlying the productivity measures frequently used by analysts.

A.5.1 U.S. Bureau of Labor Statistics (BLS)

Quarterly and annual estimates of labour productivity along with comparable measures of compensation per hour and unit labour costs are published by the BLS. Data are produced for the business sector, the non-farm business sector, non-financial corporations, the manufacturing sector and its durable and non-durable subsectors.

The BLS also produces different sets of annual multifactor productivity estimates. The multifactor productivity indices for the private business sector and the private non-farm business sectors measure the value-added output per unit of combined labour and capital inputs. Multifactor productivity indices for the manufacturing sector and its 20 constituent industries are calculated as output net of intra-industry transactions (sector output) per combined unit of capital services, labour, energy, materials and services (for more details, see BLS 1997).

The differences between the U.S. and Canadian productivity measures are the following:

1. The BLS uses two business sector concepts in its productivity estimates, both of which are different from their Canadian counterparts. Labour productivity estimates cover a business sector that is similar but not identical to the Canadian concept of the business sector. In addition to government, non-profit institutions and the imputed value of owner-occupied dwellings (all of which are excluded from the Canadian business sector), the U.S. business sector, used for labour productivity estimates, also excludes paid employees of private households. On the other hand,

[21] (…) "In order to shed light on changes in the productivity…, the Dominion Bureau of Statistics has also initiated a number of individual industry studies, mainly in the area of manufacturing. The industries to be studied were selected, in co-operation with other government departments, so as to represent a cross-section of manufacturing, including import-competing industries, export industries and typically domestic industries, and with a view to statistical feasibility and international comparability." (Dominion Bureau of Statistics 1965, forward).

U.S. multifactor productivity estimates cover only the private portion of the whole U.S. business sector as they exclude government business enterprises.

These differences are not expected to yield significant differences in terms of the coverage of the business sector between Canada and the U.S. productivity estimates. For example, government business enterprises represent a negligible portion of the U.S. business sector and their importance has been declining since the 1980s in the Canadian business sector. There are other differences, attributable to institutional factors that may, however, introduce significant differences in the coverage of the business sector in Canada and the United States. Health industries, which are part of the business sector in the United States and the government sector in Canada, are a case in point.

2. Comparisons of GDP estimates between Canada and the United States have been affected by recent changes in the definitions and the statistical methods that were incorporated into the U.S. National Accounts with the completion of their 1999 historical revisions. In the United States, two changes have been made (Parker and Grimm, 2000) to the GDP estimates. First, the method to calculate consumer price changes has been altered. Second, all software expenditures are now counted as an investment.

3. The BLS uses the Fisher Ideal index of real output for both labour and multifactor productivity indices, as does Statistics Canada.

4. The BLS uses the concept of value added only for major sectors' estimates of labour productivity (business sector and non-farm business sector) and multifactor productivity (private business sector, private non-farm business and manufacturing sector). Statistics Canada uses the concept of value added for both industries and sectors' labour productivity and multifactor productivity estimates.

   The BLS also uses the concept of sectoral output (gross output net of intra-industry transactions) for

   - labour productivity estimates of the manufacturing sector, its durable and non-durables components, its three- and four-digit industries; and,

   - multifactor productivity estimates of the manufacturing sector, its 20 two-digit industries and the 9 three- and four-digit industries that are produced. While Statistics Canada also produces comparable estimates to facilitate Canada-United States comparison of multifactor productivity, it also produces estimates of multifactor productivity based on the concept of gross output.

5. The BLS, much like Statistics Canada, makes use of the concept of hours worked.[22] Labour productivity estimates produced by Statistics Canada and the BLS both measure labour as a direct summation of hours at work. Similarly, multifactor productivity indices produced by the BLS for manufacturing industries use the same concept of labour as the labour productivity estimates.

[22] For hours worked, the BLS estimates are benchmarked on establishment surveys rather than household surveys. The establishment surveys are themselves benchmarked on administrative data from state unemployment insurance programs (Farmer and Searson 1995). Statistics Canada estimates are taken primarily but not exclusively from household surveys.

The BLS makes adjustment for labour quality only to its estimates of multifactor productivity based on value added for the private business sector and the non-farm private business sector. In this instance, the hours at work for about one thousand categories of workers are classified by their educational attainment and work experience and are aggregated using an annually chained Törnqvist index. The aggregate growth rate of labour input is therefore a weighted average of the growth rates of each type of worker where the weight assigned to a type of worker is its share of total labour compensation. Because their labour input includes labour quality changes, the BLS measures of labour and productivity are affected by these quality changes.

By contrast, Statistics Canada does not make this direct correction for labour quality. However, its method of deriving Fisher indices at the levels of sub-sectors and the business sector partially captures the adjustment of labour quality. The rate of change in hours worked by each industry is aggregated to the subsector (or sector) level using each industry's share in total labour compensation as weights. These weights will be large for industries that pay above-average wages and small for those that do not. If industries with higher wages have been growing more rapidly, this weighting system will decrease estimates of multifactor productivity relative to alternative aggregation schemes that simply take an unweighted average of the growth rates of all industries.

6. Conceptual differences between Statistics Canada and the BLS in the measurement of capital input are even more important than in the case of labour input. These differences arise from the coverage of capital and the way that detailed data on investment are aggregated by vintage and by asset type.

   BLS includes in its concept of capital, machinery and equipment, residential and non-residential structures, land and inventories at a fairly detailed level by asset type. By contrast, mainly because of paucity in the data, Statistics Canada's productivity program does not exploit the various asset types on residential and non-residential capital stock currently available from the Investment and Capital Stock Division, nor does it make use of land and inventories in the construction of the capital stock.[23]

   BLS's aggregation scheme is based on the 'relative efficiency' for aggregation by vintages and 'rental prices' for the aggregation of different types of assets. The BLS adopts 'age/efficiency' functions that decline gradually during the first few years of an asset's life, and then more rapidly as the asset ages (a concave efficiency schedule).[24] By contrast, Statistics Canada uses a geometric efficiency and

[23] Three major assets are currently available for non-residential capital stock: machinery and equipment, buildings, and engineering construction. For residential capital stock, Statistics Canada currently produces data for the following assets: singles, multiples, mobiles and cottages.

[24] BLS uses a 'hyperbolic' formula to represent the services, s_τ of a τ old asset:

$$\begin{cases} s_\tau = \dfrac{(L-\tau)}{(L-\beta\tau)} & for\ \tau < L \\ s_\tau = 0 & for\ \tau > L, \end{cases}$$

where L is asset's service life, and β is a 'shape' parameter. For $\beta = 1$, this formula yields a gross stock; for $\beta = 0$, it yields a straight line depreciation pattern and for $0 < \beta < 1$, the function declines slowly at first, and then more quickly later. BLS assumes $\beta = 0.5$ for equipment and $\beta = 0.75$ for structures. The formula was implemented assuming the U.S. Bureau of Economic Analysis' service life estimates and also assuming a discard process similar to the one used by BEA.

depreciation pattern. These differences have relatively little effect on cross-country comparisons.

As for the measurement of capital services derived from the capital stock, the BLS applies the rental price and Törnqvist aggregation techniques to detailed categories of asset types. The BLS uses a Törnqvist aggregation with rental prices formulated from Hall-Jorgenson-type tax parameters and a Jorgenson-Griliches type of internal rate of return computed using property income data from the National Income and Product Accounts.[25] In contrast, Statistics Canada sums the three components of capital stock (engineering construction, buildings, and machinery and equipment) for each industry. A Fisher index of capital input is constructed at a higher level of aggregation using capital compensation and capital stock. This methodology implicitly assumes that the capital services yielded by these three assets are equal per dollar of capital stock.

While the BLS still aggregates inputs for its multifactor productivity measures using a Törnqvist chain index, Statistics Canada has switched to the Fisher Ideal index.

A.5.3 Organisation for Economic Co-operation and Development (OECD)

The OECD publishes two sets of estimates that sometimes conflict with one another. One set is produced by the OECD Secretariat and the other by the OECD Statistics Directorate. Both estimates use imperfect measures of inputs because they are interested in cross-country comparisons and cannot get data from some countries that are required for the most precise estimates. By choosing the lowest common denominator available, they provide inaccurate estimates of the true Canadian productivity growth.

Both OECD groups use employment rather than hours-worked to calculate their estimates. This biases the Canadian results downwards.[26]

Equally important, both groups use gross and not net capital stock. It is well recognized that useful capital is net capital. This is the depreciated capital that a firm has available to it. Gross capital stock is the value of capital that was originally purchased and takes no account of the fall in value of capital that occurs over time from use of the capital in production.

Both OECD groups also incorporate another problem. Labour and capital shares of output are needed as weights for the calculation of multifactor productivity. The OECD weights are constant and do not come from Canadian data; they appear to be OECD members' averages.

[25] This implies that property income of industry i in year t is equal to the weighted sum of capital stock, $Y_{i,t} = \sum_j u_{j,i,t} K_{j,i,t} \equiv \sum_j \left(r_{i,t} + \delta_{j,i} + g_{j,i,t} \right) K_{j,i,t}$, where $Y_{i,t}$ is property income assumed to be the residual derived by subtracting labour costs from nominal value added; $K_{j,i,t}$ is the capital stock for the asset j and $u_{j,i,t}$ is the user cost of capital. Data on depreciation rate δ and the capital gain rate g are usually available, but the internal rate of return r is endogenous.

[26] See Chapter 3, "The Precision of Productivity Measures."

In addition to the above problems, the estimate of the OECD Secretariat has three problems:

- First, its measures of outputs and inputs are incompatible. Its measure of output includes owner-occupied dwellings and commercial real estate. But its measure of capital stock does not include the capital that is used for either purpose.

- Second, the measure of inputs and outputs is calculated without taking into account the underlying production structure of the economy. In other words, these estimates are calculated only at the aggregate level and suffer from the type of aggregation bias that was described above.

- Third, its measure of capital stock has been calculated arbitrarily without adequately taking into account Canadian experience. The OECD Secretariat uses an investment series taken from the National Accounts that is not used for the Canadian productivity estimates and ignores the work that has been done on depreciation and discard rates by Statistics Canada's Investment and Capital Stock series.

The OECD Statistics Directorate has created the International Sectoral Data Base (ISDB), which combines a range of data series related primarily to sectoral and industrial value added and their corresponding primary factor inputs (real GDP) used in 14 OECD member countries (OECD 1999). Based on comparable information drawn from sources released by national and international statistical agencies, the database constitutes an important basis for cross-country studies of productivity performance. Therefore, the productivity estimate of the Statistics Directorate follows procedures that are closer to those which have gained international acceptance.

The productivity estimates produced by the ISDB for the 1970 to 1997 period deals with the business sector as a whole as well as with 30 industry groupings covering all industries of 15 member countries. This source is extensively used in the international comparisons of productivity performance.

There are, however, differences between the methodology used by Statistics Canada and that of the ISDB that limit the extent to which results from these two sources may be compared:

- First, the ISDB uses a slightly different definition of the business sector. They include residential housing in their estimates of output and capital stock; Statistics Canada excludes this sector because labour inputs are missing.

- Second, the ISDB starts with individual industry data and aggregates it. However, their aggregation technique uses a Laspeyres weight for only the output, which changes every five years—the same procedure used by the National Accounts of Statistics Canada to produce GDP data. Statistics Canada productivity measures uses an annual Fisher-chained index that updates changes more frequently and is more appropriate for those industries that are experiencing rapid price changes.

- Third, the ISDB does not make any attempt to weight data from underlying industries.

- Fourth, the ISDB uses an index for capital stock that is incompatible with their output index. They choose to use a measure of capital stock which, when rebased, changes all previous growth rates. They use an index of output that does not do so. In contrast, Statistics Canada uses individual industry series for both output and capital whose past growth rates are not changed when rebasing occurs.

Despite these differences, the ISDB estimate is conceptually closer to that of Statistics Canada than that of the OECD Secretariat. At issue is the extent to which the major difference—choice of employment rather than hours worked and use of an inappropriate capital stock—can account for most of the difference between the two series.[27]

Replacing hours worked by employment accounts for most of the difference in the two series. Adding the additional change of gross rather than net capital stock leaves very little difference between the cumulative growth in the two series, despite the other differences that are still embedded in the two estimates. We conclude then that the under estimation of the Canadian productivity performance that has been produced by the Statistics Directorate is almost entirely attributable to their use of these crude measures of inputs.

A.6 Caveats

Measures of labour productivity, multifactor productivity and related measures of costs are useful in investigating the performance of Canadian industries. However, certain characteristics of the productivity and related cost data should be recognized in order to apply them appropriately to specific situations.

First, only the productivity of the business sector is measured. Because of conceptual difficulties, measures of productivity are not available for sectors of the economy, such as government, whose goods and services are not priced by the market.

Second, in several sectors where output is difficult to define, productivity measures are correspondingly weak. Examples are the business services industry, the construction industry and the financial services sector, where output is often an imputed value of labour and other inputs. Thus, the productivity and costs measures for these sectors should be interpreted with caution.

Third, the capital input used in the multifactor productivity framework does not account for land, inventories and natural resources stock, public capital stock and research and development (R&D). Some experimental studies have concluded that natural capital stock, public capital and R&D contribute significantly to multifactor productivity growth.[28] However, these types of inputs pose important challenges in terms of measurement of the quantities and price of services. Nonetheless, as part of an effort to improve the coverage of capital and, accordingly, to increase the comparability between Canadian and U.S. productivity measures, the productivity program has given a priority to estimating land and inventories.

Fourth, measures of productivity account only for resources used in the production process. Unemployed resources available in the economy, which indicate the extent to which the economy is close to its potential capacity, are therefore excluded from the productivity estimates. Nonetheless, comparisons of labour productivity growth and the growth of GDP per capita help to indicate the consequences of not fully employing all labour resources.[29]

Fifth, resources engaged in the production process may not be fully employed, as is often the case in economic downturns. Labour hoarding is a classical example: in response to decreasing demand for its product, an industry may not lay off its employees

[27] See Chapter 3, "The Precision of Productivity Measures."
[28] See Harchaoui (1997), Diaz and Harchaoui (1997) and Mamuneas and Nadiri (1996).
[29] See Chapter 4 on the Canada–United States comparison for a discussion of these issues.

for various reasons such as separation costs and the cost of training new employees should operations expand later on.

A partial adjustment is made to take into account the capacity utilization rate of capital by using the compensation of capital rather than the user cost of capital (Berndt and Fuss 1986). However, at best, this approach only partially dampens the cyclical fluctuations of the productivity growth rates. Since the cyclical fluctuations generally shown by the standard productivity growth measures are often used to make inferences about long-term economic performance, users should be cautious about inferring long-run trends from changes on a yearly basis. To reduce the influence of the cycle on economic performance, users are encouraged to consider a peak-to-peak or a trough-to-trough analysis of productivity growth rates.

A.7 Concluding remarks

This appendix has discussed the development of the Statistics Canada productivity measures program produced for the Canadian business sector and its major constituents (subsectors and industries). It has touched on advances in the literature on productivity measurement and described how these advances have led Statistics Canada to improve the methods it uses and to develop new data series consistent with these advances.

Some further refinements are presently being explored. These advances deal with the quality measurement of the inputs and a broader coverage for the concept of capital that includes land, inventories and exhaustible resources stocks. There are also new lines of research in the productivity front that are worth investigating in the near future. Among these are studies using firm or establishment level data,[30] studies that relax the assumption of constant returns to scale underlying the multifactor productivity framework,[31] and studies designed to expand the scope of productivity measurement to include environmental considerations.

References

Berndt, E.R. and M.A. Fuss 1986. “Productivity measurement with adjustments for variations in capacity utilization and other forms of temporary equilibrium.” *Journal of Econometrics.* 33: 7-29.

Cas, A. and T.K. Rymes. 1991. *On Concepts and Measures of Multifactor Productivity.* Cambridge: Cambridge University Press.

Diaz, A. and T.M. Harchaoui. 1997. “Accounting for Exhaustible Resources in the Canadian System of National Accounts: Flows, Stocks and Productivity Measures.” *Review of Income and Wealth.* 43: 465-485.

Diewert, W. E. 1976. “Exact and Superlative Index Numbers.” *Journal of Econometrics.* 4: 115-145.

Domar, E.D. 1961. “On the Measurement of Technical Change.” *Economic Journal. 71*, 204: 709-729.

[30] See Chapter 5 in this publication.
[31] See Chapter 8 in this publication.

Dominion Bureau of Statistics. 1965. *Indexes of Output Per Person Employed and Per Man-Hour in Canada, Commercial Nonagricultural Industries, 1947-63.* Statistics Canada Catalogue no.14-501. Ottawa: Minister responsible for Statistics Canada. Occasional.

Durand R. 1996. "Canadian Input-output-based Multifactor Productivity Accounts." *Economic Systems Research.* 8: 367-389.

———. 1994. "Le programme de productivité multifactorielle de Statistique Canada." *L'actualité économique.* 69: 313-330.

Farmer, T.E. and M.A. Searson. 1995. "Use of Administrative Records in the Bureau of Labor Statistics." *Covered Employment and Wages (Es-202) Program.* 1995 Bureau of the Census Annual Research Conference. U.S. Bureau of Labor Statistics. Washington: The U.S. Government Printing Office.

Gollop, F.M. 1979. "Accounting for Intermediate Input: The Link Between Sectoral and Aggregate Measures of Productivity." *Measurement and Interpretation of Productivity.* Washington: National Academy of Sciences. *318-333.*

Griliches, Z. 1980. "R&D and the Productivity Slowdown." *American Economic Review.* 70: 343-348.

Harchaoui, T.M. 1997. "Le capital public au Canada: évolution historique et externalités." *L'économétrie appliquée, L'actualité économique.* C.Gouriéroux and C. Montmarquette (eds). 73: 395-421.

Jackson, C. 1996. "The Effect of Rebasing on GDP." *National Economic and Financial Accounts.* Second Quarter. Statistics Canada Catalogue no. 13-001-XPB. Ottawa: Minister responsible for Statistics Canada.

Jorgenson, D.W. 1990. "Productivity and Economic Growth." *Fifty Years of Economic Measurement.* E. R. Berndt and J.E. Triplett (eds). NBER. *Studies in Income and Wealth.* 54: 19-118.

Jorgenson, D.W. and Z. Griliches. 1967. "The Explanation of Productivity Change." *Review of Economic Studies.* 99: 249-283.

Jorgenson, D.W. and K.-Y. Yun. 1991. *Tax Reform and the Cost of Capital.* New York: Oxford University Press.

Lal, K. 1998. *The 1997 Historical Revision of the Canadian System of National Accounts: Records of Changes in Classification of Sectors and Transactions, Concepts and Methodology.* Statistics Canada Catalogue no. 13N0005XPE. March.Ottawa: Minister responsible for Statistics Canada.

Mamuneas, T and I. Nadiri. 1996. "Public R&D Policies and Cost Behavior of the U.S. Manufacturing Industries." *Journal of Public Economics.* 63: 57-81.

Mirotchie, M. 1996. "Methodology Used to Produce Advance Estimates of Multifactor Productivity Indices for the Canadian Aggregate Business Sector." *Aggregate Productivity Measures.* Statistics Canada Catalogue no.15-204-XPE. Ottawa: Minister responsible for Statistics Canada.

Organisation for Economic Co-operation and Development (OECD). 1999. *International Sectoral Database 98, User's Guide.* Statistics Directorate. Paris: OECD.

Parker, R. and B. Grimm. 2000. "Software Prices and Real Output: Recent Developments at the Bureau of Economic Analysis." Paper presented at the National Bureau of Economic Research on Technological Change and Productivity Measurement. April 17. Bureau of Economic Analysis. U.S. Department of Commerce.

Rees, A. 1980. "Improving Productivity Measurement." *American Economic Review.* 70: 340-342.

Rymes, T.K. 1972. "The Measurement of Capital and Total Factor Productivity in the Context of the Cambridge Theory of Capital." *Review of Income and Wealth.* 18: 79-108.

Solow, R.M. 1958. "'Technical Change and the Aggregate Production Function." *Review of Economics and Statistics* 39: 312-320.

Statistics Canada. 1975. *A Guide to the National Income and Expenditure Accounts: Definitions,Concepts, Sources and Methods.* Catalogue no. 13-549E, vol.3. National Income and Expenditure Accounts. Ottawa: Minister responsible for Statistics Canada. Occasional.

———. 1999. *Canadian Net Capital Stock Estimates and Depreciation Profiles: A Comparison Between the Existing Series and a Test Series Using the U.S. (BEA) Methodology.* March. Investment and Capital Stock Division.Ottawa.

U.S. Bureau of Labor Statistics. 1997. *BLS Handbook of Methods.* Bulletin 2490. Washington: The U.S. Government Printing Office.

Wolfe, E.N. 1991. "Dynamic of Growth in Input-output Analysis." *Technology and Productivity, the Challenge for Economic Policy.* Paris: OECD.565-574.