Quadratic modelling of Canada's Baby Boom

Overview

In this lesson, students will learn about applications of the vertex form [$\mathbf{y = a(x - h)^2 + k}$] of a quadratic equation. Students will extract data on births in Canada from Statistics Canada's E-STAT database and import them into a statistical software program. Within the software program, students will model a quadratic equation. By adjusting the values of the **a**, **h**, and **k** parameters to maximize the fit of the parabola to the Canadian Baby Boom data, students will gain a greater understanding of the purpose of the parameters in the vertex form of a quadratic equation.

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Objectives

- Through investigation, determine the relationships between the graphs and the equations of quadratic functions
- Through investigation, determine the basic properties of quadratic functions
- Identify the effect of simple transformations
- Explain the role of **a**, **h**, and **k** in the vertex form [$\mathbf{y = a(x - h)^2 + k}$] of the quadratic equation representing a parabola
- Collect secondary data that may be represented by a quadratic function
- Fit the equation of a quadratic function to a scatter plot using an informal process

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Suggested grade levels and subject areas

Grades 10 and 11
Mathematics

Duration

One to two 75 minute periods

Materials

Computers with Internet access and statistical software
Computer projector
E-STAT account
E-STAT instructions
Software instructions:

- Generic
- Dynamic statistical Fathom software (PDF)

Student worksheet
Student worksheet – Teacher version

Prior knowledge

Vertex form of quadratic equation [$\mathbf{y = a(x - h)^2 + k}$]
Basic knowledge of E-STAT and statistical software

Classroom instructions

1. Discuss important properties of the vertex form of the quadratic equation as a review.
2. Using the computer projector, demonstrate the important features of E-STAT (http://estat.statcan.ca).
3. Hold a brief class discussion on the topic of the Baby Boom to assess students' prior knowledge and share information on the topic.
4. Distribute the student instructions and worksheet and have students complete the lesson independently or in pairs.

Enrichment

Have students repeat this process for a later time period to see if there is a significant Baby Boom Echo. Ask them to compare the shapes of the two curves, find the peak year of the Baby Boom Echo, find the period between the two peaks, and interpret this period of time.

Have students repeat this process for their province or territory instead of Canada as a whole. Ask them to compare the shapes of the graphs. If the shape of the graph for the province or territory is different from the shape of the graph of Canada as a whole, ask students to research reasons for this difference.

Have students import the data for the Baby Boom and the Baby Boom Echo into a graphing calculator or spreadsheet software to perform quadratic regression analysis. Have the students compare their curve of best fit for the data with the regression analysis.

Challenge your students to search on the E-STAT CANSIM database to find other time series data (among millions of time series) that can be modelled by a quadratic function. They can import these data into a statistical software program and attempt to plot quadratic functions to fit the data.

If students find an E-STAT time series that can be modelled well by a quadratic function, please e-mail us at eduliaison@statcan.ca.

Evaluation

Students can be informally assessed on their work habits and computer skills throughout this activity. They can be formally assessed via the worksheet, which can be marked using a marking scheme of the teacher's choice.

Student instructions
Quadratic modelling of Canada's Baby Boom

Background information

A graph of the human population rarely approximates the shape of a parabola with any accuracy. However, underlying conditions occasionally lead to population trends that closely follow a parabolic shape over specific time periods. The Baby Boom occurred right after World War II. Canadian soldiers returning in great numbers after the war started having children within a few years after returning to Canada, which resulted in a huge increase in the number of births. The number of births increased for several years and then began to decline. In this activity, you will determine if a parabola can be a useful model for the number of births per year for this post-war Baby Boom period.

E-STAT instructions

1. Go to the E-STAT website (http://estat.statcan.ca).
2. Select **English**.
3. Click **Search CANSIM** on the left sidebar.
4. Type **053-0001** in the Search box and click on the **Search** button to retrieve **Table 053-0001 – Vital statistics, births, deaths and marriages, quarterly (number)**.
5. On the **Subset selection** page:
   a. Under **Geography**, select **Canada**
   b. Under **Estimates**, select **Births**
   c. Under **From**, select **Jan 1950**
   d. Under **To**, select **Dec 1968**
7. Click on the **Retrieve as individual Time Series** button.
8. On the **Output specification** page, select either "**Plain text: Table, time as rows**", "**HTML, table: Time as rows**", or "**CSV: Time as rows**". The output selected will depend on your software. Ask your teacher which format works best with your school's software.
9. Click on the **Manipulate data** button at the bottom of the screen.
10. Under **Frequency of output data will be:**, then under **Converted to:**, select **Annual (sum).**
    *The data as retrieved were quarterly (every three months). By selecting **Annual (sum)**, you are adding the quarterly data together to result in an annual total.*
11. Click on the **Retrieve now** button.

Generic statistical software instructions

Note: Specific instructions for dynamic statistical software (e.g. Fathom) are also provided at http://teacherweb.com/on/statistics/math/BabyBoom-FathomVersion-Instructions.pdf.

1. Import your data into your statistical software program.
2. Save your file as **Baby Boom**.
3. Create a **scatter plot** with **Year** on the **x-axis** and **Number of births** on the **y-axis**.
4. Plot the following function on your scatter plot:
   Number of births = $a(\text{Year} - h)^2 + k$

******* Go to your worksheet and answer questions #1 to 11. *******

5. Alter the values of **a**, **h**, and **k** until your function closely approximates the data in the scatter plot.
   ******* Go to your worksheet and answer questions #12 to 18.**

Student worksheet
Quadratic modelling of Canada's Baby Boom

1. In general, what does **h** represent?

2. Specific to this dataset, what does **h** represent?

3. By inspection of your scatter graph, what is a reasonable approximation of **h**?

4. In general, what does **k** represent?

5. Specific to this dataset, what does **k** represent?

6. By inspection of your scatter graph, what is a reasonable approximation of **k**?

7. What is the difference in the parabola between a quadratic equation with a positive **a** value and a quadratic equation with a negative **a** value?

8. What happens if **a** is zero? Discuss in terms of both the equation and the resulting graph.

9. If a quadratic equation has a positive **a** value, will the vertex of the parabola be a maximum or a minimum? Why?

10. If a quadratic equation has a negative **a** value, will the vertex of the parabola be a maximum or a minimum? Why?

11. How does changing the value of **a** affect the shape of a parabola?

*******Return to your instruction sheet and complete the final step before answering questions #12 to 18*******

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12. What is your best value for **a**?

13. What is your best value for **h**?

14. What is your best value for **k**?

15. Write your equation here in the form $y = a(x-h)^2 + k$.

16. How are h and k related?

17. What is the vertex of your graph? What does this mean in terms of this dataset?

18. Paste your graph here.

Student worksheet — Teacher version
Quadratic modelling of Canada's Baby Boom

1. In general, what does **h** represent?

   h *represents the x value at which the optimum (maximum or minimum) y value occurs. In other words,* ***h*** *is the x co-ordinate of the vertex.*

2. Specific to this dataset, what does **h** represent?

   For this dataset, ***h*** *represents the year in which the maximum number of births occurred.*

3. By inspection of your scatter graph, what is a reasonable approximation of **h**?

   By inspection, ***h*** *appears to be approximately 1960.*

4. In general, what does **k** represent?

   k *represents the optimum (maximum or minimum) y value. In other words,* ***k*** *is the y co-ordinate of the vertex.*

5. Specific to this dataset, what does **k** represent?

   For this dataset, ***k*** *represents the maximum number of births.*

6. By inspection of your scatter graph, what is a reasonable approximation of **k**?

   By inspection, ***k*** *appears to be approximately 480,000.*

7. What is the difference in the parabola for a quadratic equation with a positive **a** value and a quadratic equation with a negative **a** value?

   If the ***a*** *value is positive, the parabola opens up. If the* ***a*** *value is negative, the parabola opens down.*

8. What happens if **a** is zero? Discuss in terms of both the equation and the resulting graph.

   If ***a*** *is zero, the equation becomes y = k. This means that the resulting graph is a horizontal line at y = k.*

9. If a quadratic equation has a positive **a** value, will the vertex of the parabola be a maximum or a minimum? Why?

   If the quadratic equation has a positive ***a*** *value, the vertex of the parabola will be a minimum since the graph opens up.*

10. If a quadratic equation has a negative **a** value, will the vertex of the parabola be a maximum or a minimum? Why?

    *If a quadratic equation has a negative **a** value, the vertex of the parabola will be a maximum since the graph opens down.*

11. How does changing the value of **a** affect the shape of a parabola?

    *For positive **a** values, as **a** increases, the graph is stretched vertically (becomes narrower). When **a** is zero, the graph is a horizontal line (y = k value). For negative **a** values, the graph is stretched vertically (becomes narrower) as the value of **a** decreases (becomes more negative). Essentially, as the absolute value of **a** increases, the graph becomes narrower.*

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12. What is your best value for **a**?

    *Answers vary. In this example, **a** = -1,545.*

13. What is your best value for **h**?

    *Answers vary. In this example, **h** = 1959.*

14. What is your best value for **k**?

    *Answers vary. In this example, **k** = 479,600.*

15. Write your equation here in the form $\mathbf{y = a(x - h)^2 + k}$.

    $y = -1{,}545(x - 1959)^2 + 479{,}600$

16. How are **h** and **k** related?

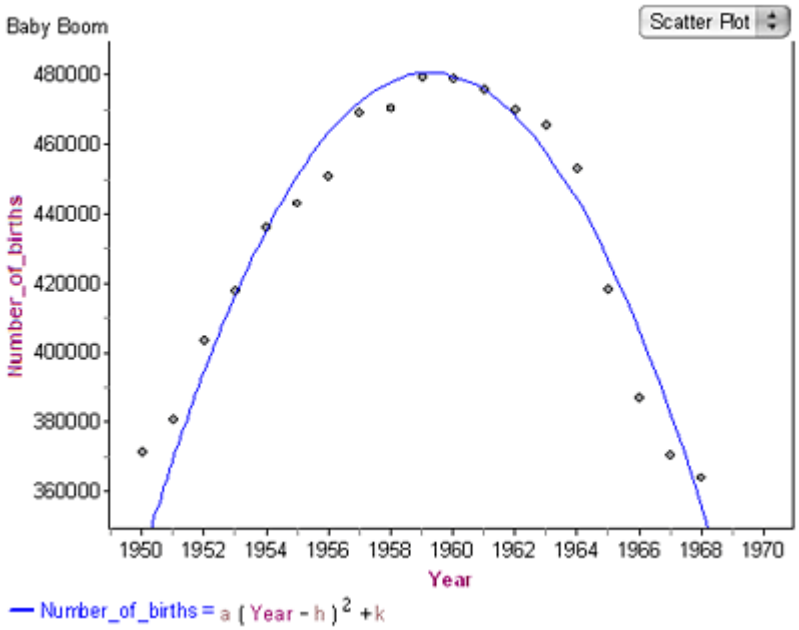
    ***h** and **k** are the x and y values, respectively, of the vertex. That is, the vertex is the point (**h**, **k**).*

17. What is the vertex of your graph? What does this mean in terms of this dataset?

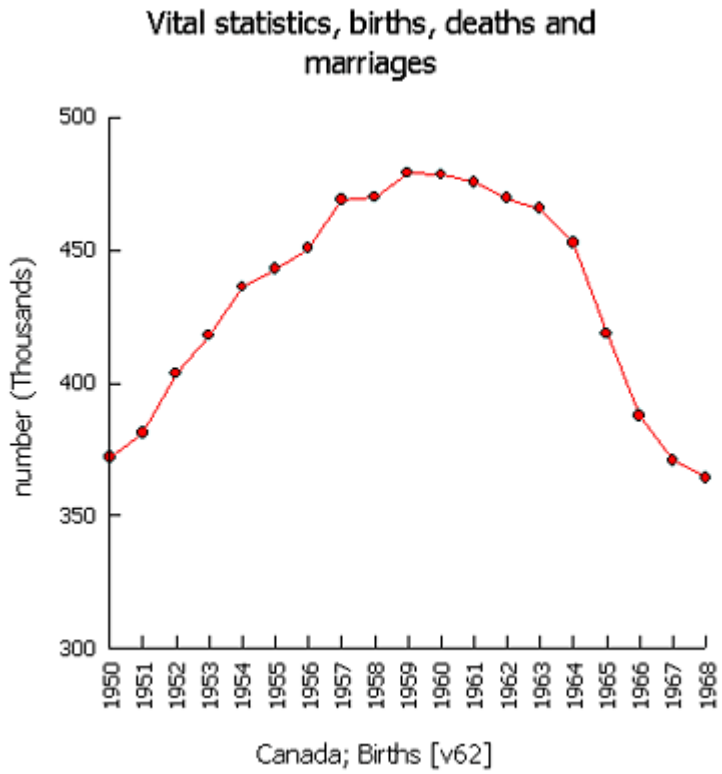
    Answers vary. In this example, the vertex of the graph is (1959, 479,600). This means that the highest number of births (479,600) occurred in the year 1959.

18. Paste your graph here.

    ***Fathom** graph*
    (Fathom is licensed by the Ministry of Education and used by schools in some provinces. Providing the Fathom format is in no way an endorsement or recommendation of the Fathom software by Statistics Canada.)



***E-STAT** graph*
To generate the graph below, select Line graph with symbols in step 8 of [E-STAT instructions](#).



Source: Statistics Canada. Table 053-0001 - Vital statistics, births, deaths and marriages, computed annual total (number) (graph), CANSIM (database), Using E-STAT (distributor). http://estat.statcan.ca/cgi-win/cnsmcgi.exe?Lang=E&ESTATFile=EStat\English\CII_1_E.htm&RootDir=ESTAT/ (accessed: April 10, 2008)